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Engineering Aspect of Automobile Accident Reconstruction Using Computer Simulation

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ABSTRACT: Using engineering principles including laws of conservation of energy and momentum, automobile accidents involving two or more cars can be reconstructed. The information obtained in a field investigation and the police report is used as input to a microcomputer. Execution of a program developed specifically for such purposes yields a sequence of events that must have taken place resulting in the accident. The program employs the post-impact information and pre-impact headings of the vehicles to determine the pre-impact velocities of the cars. Computer-aided investigation is of paramount interest to law enforcement agencies, insurance companies, and trial lawyers.

KEYWORDS: engineering, automobiles, accidents, computers, automobile accident reconstruction, computer simulation, computer graphics, accident analysis, forensic engineering

This paper presents a brief description of the mechanics involved in the collision of two moving vehicles. For clarity, many mathematical details are left out; interested readers are referred to standard references for a thorough treatment [*1-6*].

Consider a two-car crash as shown in Fig. 1. Vehicles represented by Cubicles 1 and 2 were moving with velocities V_1 and V_2 in specified directions as indicated by arrows in Fig. 1. Point A is the point of impact. After the collision, Vehicle 1 traveled a distance L_1 before coming to rest at Point A_1 and Vehicle 2 traveled a distance L_2 and came to rest at A_2. Let the velocities of the vehicles right after impact be V_1 and V_2. By virtue of the law of conservation of energy, the energy dissipated by each vehicle in sliding the distances L_1 and L_2 must be equal to the kinetic energies of the respective vehicles just after the impact. Whence

$$(\mu W_1)L_1 = \frac{1}{2}\frac{W_1}{g}V_1^2$$

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or, after solving for V_1,

$$V_1 = \sqrt{2\mu g L_1}$$

$$(\mu W_2)L_2 = \frac{1}{2}\frac{W_2}{g}V_2^2 \text{ or } V_2 = \sqrt{2\mu g L_2}$$

where μ is the coefficient of friction, W_1 and W_2 are the weight of the respective vehicles, and g is the gravitational acceleration which is equal to 9.8 m/s^2 (32 ft/s^2).[3] Obviously, if this is all there is to it, one does not need a computer; a hand calculator will suffice. In reality Vehicle 1 did not move on a straight line from A to A_1, but it took a curved path which is termed "trajectory." Also, the motion of each vehicle after impact was composed of a simultaneous translation and rotation (spinning) along the trajectory. In general, both linear and angular velocities of a spinned out car reduce because of ground friction. However, the decelerations are not uniform but the linear and angular velocities of the vehicles decrease alternately as the heading direction changes with reference to the direction of its center of gravity, at least for free-rolling wheels. To account for these complicated variations, an arithmetic solution is obviously impossible. Therefore, one has to solve the differential equation of motion numerically by computer because it is usually a complicated one and does not have closed form solution [*7,8*]. This procedure is known as a "spinout analysis." The treatment we have shown in using linear translation motion is a simplification for the purpose of demonstration.

It is well-known that through the principle of balance of momentum, the post-impact velocities are related to pre-impact velocities of the vehicles just before the accident. Let us assume the orientation of vehicles before the accident referenced to a fixed axis be α_1 and α_2 and after the impact be θ_1 and θ_2 and the velocities of the vehicles before the accident be v_1 and v_2 and im-

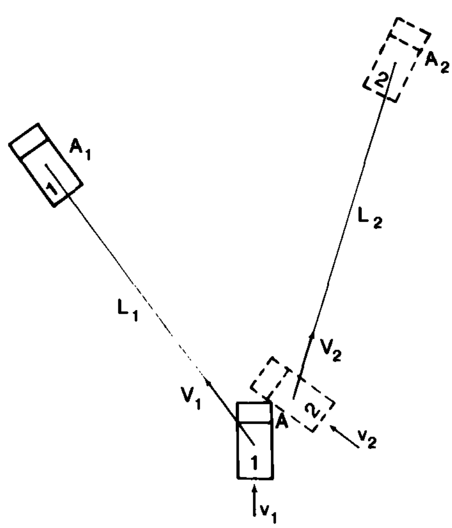

FIG. 1—*Schematic diagram showing two vehicles, 1 and 2. Point* A *is the impact point. Points* A_1 *and* A_2 *are the resting position of the two cars, respectively.*

[3]The original calculations were in the inch/pound system.

mediately after the impact be V_1 and V_2 for Vehicles 1 and 2, respectively. The law of balance of momentum requires that

$$m_1 V_1 \mathrm{Cos}\,\theta_1 + m_2 V_2 \mathrm{Cos}\,\theta_2 = m_1 v_1 \mathrm{Cos}\,\alpha_1 + m_2 v_2 \mathrm{Cos}\,\alpha_2$$

$$m_1 V_1 \mathrm{Sin}\,\theta_1 + m_2 V_2 \mathrm{Sin}\,\theta_2 = m_1 v_1 \mathrm{Sin}\,\alpha_1 + m_2 v_2 \mathrm{Sin}\,\alpha_2$$

Solving these two equations simultaneously the pre-impact velocities v_1 and v_2 are determined.

In reality, the impact alignment of engaging vehicles is seldom central, and thus, in addition, angular momenta would also be created. Further complications are caused by the crumbling of sheet metals as two vehicles push each other, during which, both distribution and intensity of interacting forces are ever changing. In turn, the principal forces of impact would change in directions and in amount. All these and other factors can and do affect the momentum and energy transfers and must be accounted for in a complete analysis. Therefore, momentum equations shown above are again an oversimplified version for illustration purposes only.

In principle, even during the impact, the kinetic and strain energies are conserved. It would be appropriate if there was a good way to keep track of the temperature and dynamic deformation and constitutive properties of materials involved. This is not usually practical except for measuring the permanent set at various locations of the involved vehicles. Fortunately, the energy that is responsible for damages depicted in a post-impact measurement can account for the major part of the energy dissipation. Research results showed that [*1*] the energy in a non-oblique dent can be accurately estimated by a quadratic equation in terms of the depths of identation over a width L as follows:

$$\text{Energy} = \int_0^L (k_1 + k_2 c + k_3 c^2)\,dL$$

where k_1, k_2, and k_3 are the stiffness coefficients of the parts of the vehicle damaged and c is the depth of indentation as a function of location.

In reality, the crash is usually somewhat oblique to the surface and the tangential as well as normal stiffnesses of the vehicle body must enter into the formulation. The above expression is thus once more a simplified illustration.

As it turns out, the pre-impact kinetic energy minus the post-impact energy of the involved vehicles should be comparable to the crash energy as predicted by the damage information. The energy that is unaccounted for should be the vibratory energy that is dissipated into heat during the crash. In other words, the moment and energy calculation should give consistent results up to a reasonable degree of certainty.

The use of energy and momentum conservation along with the consistency check with the available damage information are given in a schematic chart in Fig. 2. Note that because of lack of complete information or invalid information, it is crucial to perform consistency checks.

Comparison with Test Results

Similar to all analytical procedures, the accident reconstruction methods need to be verified for their fidelity. If the method is programmed for a computer, such verification is known as a *validation*. The validation is carried out with full-scale tests. The tests were staged crashes with instrumental vehicles under controlled conditions. The results of one set of validation for velocity change from energy calculations are shown in Fig. 3. It is evident that the calculated and measured velocity changes are highly correlated [*5,6,9*], and the agreement appears to be good.

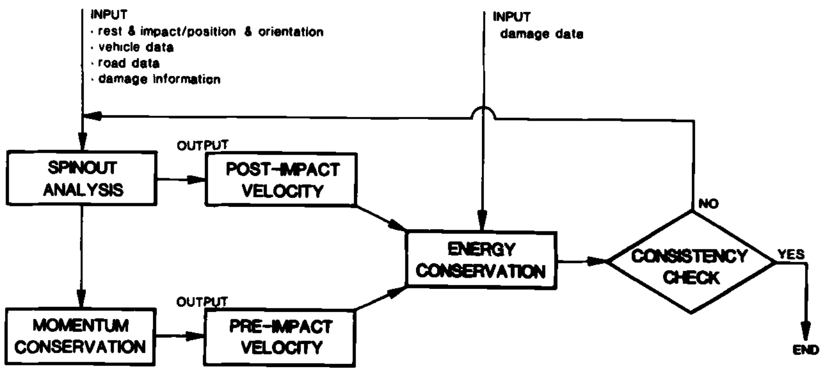

FIG. 2—*Schematic diagram of velocity calculation and consistency checks.*

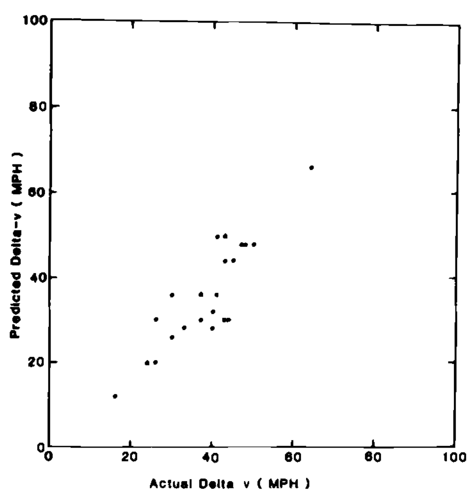

FIG. 3—*Plot of predicted and actual velocity change (Delta-v).*

Application to a Real Automobile Accident (a Case Study)

The accident took place at approximately 4:30 p.m., on a two-lane undivided rural road, heading in a north-south direction. Slightly southern to the site of the accident the road has a gentle downgrade, crossing a railroad track and then over a small creek. It was dusk and rainy but the roadway was snow-covered and snow and ice covered the shoulders of the roadway and beyond. The posted speed limit was 56 km/h (35 mph). Neither driver can recall any details of the accident, which makes an interesting and rather difficult assignment for forensic scientists or accident investigators. They need a sophisticated analytical tool to come up with a theory or opinion as to how the accident most likely occurred and who was at fault.

The following are the circumstances of the accident. Vehicle 1 (a mini car) was traveling south, approaching the railroad tracks. Apparently the driver decided to slow down to avoid the bump while crossing them. This caused Vehicle 1 to skid on the slippery roadway by some 75° to face the southeastern direction just before the tracks. Vehicle 2 (a pickup truck), which was following Vehicle 1, hit Vehicle 1 obliquely. The impact threw Vehicle 1 along a southeasterly direction about 26 m (85 ft) into a creek. Vehicle 2 came to rest approximately 17 m (55 ft) down the road in an essentially southern direction.

We have simulated this accident in a computer through a program which we have developed for crash analysis.

The input to this program consists of four sets of data. The first set of data includes the physical characteristics of the vehicles: weights, size, and distribution of weights over the vehicles, and so forth. These numbers are available for most vehicles and are tabulated in several National Highway Traffic Safety Administration (NHTSA) manuals such as Ref *1*. The information for the vehicles involved are given in Table 1. The information for some trucks, new, or European vehicles not listed in the above manual can be obtained from the manufacturers.

The second set of data includes the final resting positions and orientations of the vehicles. This information is usually obtainable from the police accident report.

The third set of data involves the actual collision, including the locations and orientations of the vehicles at impact and the heading of the vehicles just before impact. In this case this information is clearly defined. Vehicle 1 was skidding to an angle of about 75° from its forward direction and Vehicle 2 was moving in a southerly direction, although not exactly due south since it, too, started to skid.

The last set of data covers the coefficients of friction and wheel locking factors for each vehicle. The coefficients of friction are numbers measuring the degree of friction between the tires and the surface of the roadway, or in other words, how much energy must be dissipated for a vehicle to slide over the roadway. Wheel locking factors relate to whether or not the vehicle wheels are free to rotate because of braking or have been damaged and cannot rotate. They also relate to the amount of rotation in the spinout of the vehicle after impact. These numbers are tabulated for varying conditions in NHTSA manuals such as Ref *1*.

Our approach in this case was as follows. From a study of impact and rest locations and orientation, the speed of each vehicle immediately following impact was calculated. This is based on the spinout analysis through numerical solution of the differential equations of the vehicle trajectories. Once the post-impact speeds were determined, the law of conservation of momentum was applied to obtain the pre-impact speeds. Since the directions of motion were given to the computer as input data, the program logic checks for the possibility that these directions were inconsistent with the law of conservation of energy or with the post-impact speeds. Calculations based on the energy dissipated in the damage of the vehicles were also done, and were used to check that energy was conserved and that the changes in vehicle speeds were consistent with the damage energy information.

The coefficient of friction for snow-covered roads was given by Baker [*10*] to be 0.10 to 0.20 for loose snow and 0.35 to 0.55 for packed snow. Giving benefit to the adversary side and using the average of low limits for semi-packed snow, we choose a coefficient of friction of $(0.10 + 0.35)/2 = 0.23$ for the roadway. Since Vehicle 2 traveled only on the roadway we use 0.23 for its

TABLE 1—*Physical characteristics of the vehicles.*[a]

	Mini Car (Vehicle 1)	Pickup Truck (Vehicle 2)
Weight	2469 lbs	3579 lbs
Radius of gryation squared	2006 in.^{2}	2469 in.^{2}
Wheelbase	81 in.	133 in.
Length	13.3 ft	17.4 ft
Width	5.0 ft	6.4 ft

[a] 1 lb = 0.45 kg, 1 in.^{2} = 6.45 cm^2, 1 in. = 25.4 mm, and 1 ft = 0.3 m.

coefficient of friction. For Vehicle 1, which traveled partly on the roadway and partly on the icy shoulder and roadside, we use a composite figure of weighted average of the respective coefficients. The method of calculation was as follows. The total distance traveled by Vehicle 1 after impact was known to be 26 m (85 ft), of which 18 m (60 ft) was on snowy roadway and 7.6 m (25 ft) was on the roadside. According to Baker [10], the coefficient of (smooth) ice with rubber ranges from 0.07 to 0.20. We use a conservative figure of 0.07 again giving the adversary the benefit of doubt. The resulting coefficient of friction for Vehicle 1 is

$$\frac{(60) \times 0.23 + (25) \times 0.07}{85} = 0.18$$

The locking factors for both vehicles were taken to be 1.0, since both vehicles had skidded.

Using the above data, the accident was reconstructed. The results of the reconstruction stipulate that Vehicle 1 must be almost at rest before the impact, while Vehicle 2 was moving at a speed somewhat above 48 km/h (30 mph).

Under the given road conditions, ($\mu = 0.23$) calculated that if Vehicle 2 applied its brakes 15 m (50 ft) before impact, it had to be moving 56 km/h (35 mph). If the brakes were applied 30.5 m (100 ft) before impact, the calculated speed would be 64 km/h (40 mph). In either event we assumed a conservative case, that is, that Vehicle 2 skidded immediately upon application of the brakes. If this is not true and Vehicle 2 had not skidded immediately, the calculated speeds would be higher, in the range of 64 to 80.5 km/h (40 to 50 mph). The increase in speed results from the fact that a braking vehicle slows down more than a skidding vehicle.

The computer program can graphically illustrate the history of the two vehicles, before, during, and after impact, and, in fact, one can get a graphics output giving the relative distance and orientation of vehicles with respect to each other and the roads. Figure 4 shows some of such graphic output at appropriate time intervals. Figure 5 shows a composite figure that is obtained by superimposing all graphs shown in Fig. 4 on a single graph. These frames can clearly show where the vehicles were at any given instance of time. The results show that Vehicle 2 (the pickup truck) was moving at a speed of 64 km/h (40 mph) when the driver saw Vehicle 1 skidding and turning broadside to the roadway. In our opinion, a speed of 64 km/h (40 mph) on a snow-covered road posted at 56 km/h (35 mph), on a dark, rainy evening was an excessive speed. Separate calculation showed that, had the driver of Vehicle 2 been going slower, or not been following so closely behind Vehicle 1, the accident could have been avoided.

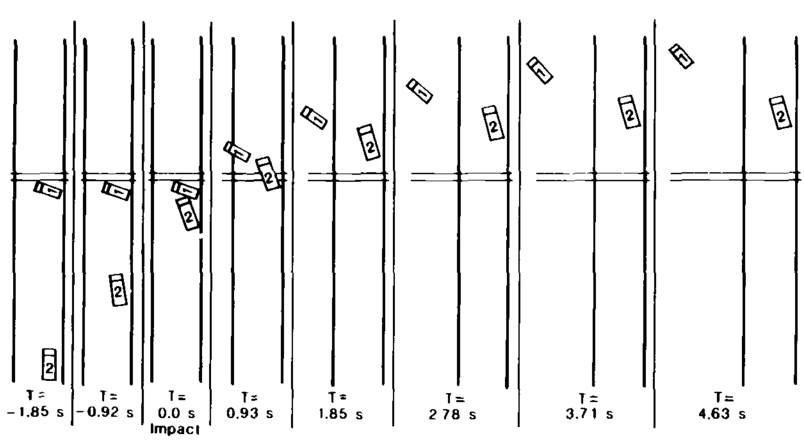

FIG. 4—*Computer graphics showing relative position and orientation of the two vehicles at several instances. T = 0 s represents the instant of impact while negative time signifies the pre-impact time.*

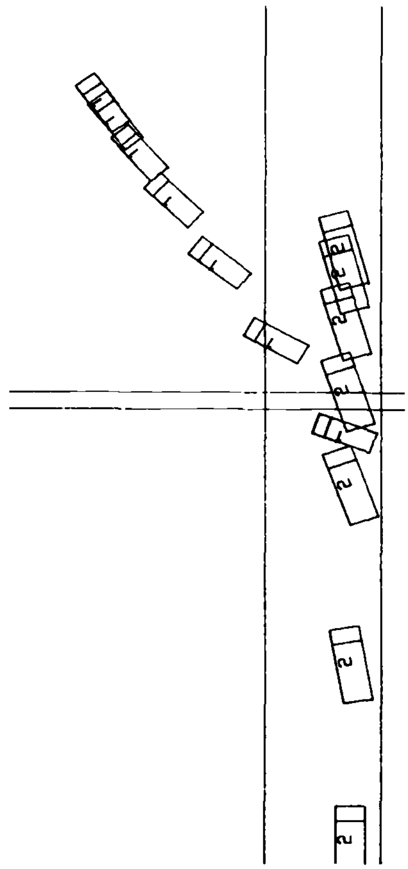

FIG. 5—*Composite figure obtained by superimposing the frames shown in Fig. 4.*

Conclusions

In conclusion, we have demonstrated, through a real case, the capability of our computer programs for accident reconstruction. This is an engineering approach based on scientific principles and certainly can help lawyers and forensic pathologists in determining the exact sequence of events that must have taken place in the accident. Because of the high number of everchanging variables, it is impossible to cope with such complex problems and perform a parametric study without a computer. We believe that computer graphics can also be used effectively during a trial to depict the accident scenario more vividly than a verbal discussion.

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